A1

VERSION CODE

ELIGIBILITY/COMPETITIVE EXAM 2024
PAPER-2
Total Number of Questions: 100
Maximum Marks : 200

Serial Number :

MENTION YOUR REGISTER NUMBER

Subject: MATHEMATICS

INSTRUCTIONS FOR CANDIDATES

DOs:

1. This question booklet is issued to you at **9.55 a.m.** by the room invigilator.
2. Check whether the Register Number has been entered and shaded in the respective circles on the OMR answer sheet.
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5. Compulsorily sign at the bottom portion of the OMR answer sheet in the space provided.

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IMPORTANT INSTRUCTIONS TO CANDIDATES

1. In case of usage of signs and symbols in the questions, the regular textbook connotation should be considered unless stated otherwise.
2. This question booklet contains **100** questions and each question will have one statement and four different options / responses & out of which you have to choose one correct answer.
3. At **10.00 a.m.** remove the paper seal of this question booklet and check that this booklet does not have any unprinted or torn or missing pages or items etc., if so, get it replaced by a complete test booklet within **5** minutes of the commencement of exam. Read each item and start answering on the OMR answer sheet.
4. Completely **darken / shade** the relevant circle with a **blue or black ink ballpoint pen against the question number on the OMR answer sheet.**

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① ● ③ ④	⊗ ② ③ ④ ① ② ③ ✓ ① ● ● ④
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5. Please note that even a minute unintended ink dot on the OMR answer sheet will also be recognized and recorded by the scanner. Therefore, avoid multiple markings of any kind on the OMR answer sheet.
6. Use the space provided on each page of the question booklet for Rough Work. Do not use the OMR answer sheet for the same.
7. **Once the last Bell rings at 1.00 P.M.,** stop writing on the OMR answer sheet and hand over the **OMR answer sheet** to the room invigilator as it is.
8. After separating the top sheet (Office copy), the invigilator will return the bottom sheet replica (Candidate's copy) to you.
9. All questions carry equal marks.
10. Use of Mobile Phones, Calculators and other Electronic / Communication gadgets of any kind is prohibited inside the Examination venue.

1. If $x > 0$ and if y is an arbitrary real number, then there is a positive integer n such that _____

(1) $nx > y$ (2) $nx < y$

(3) $nx = y$ (4) $nx = -y$

2. The sequence $\{(-1)^n\}_{n=1}^{\infty}$ is _____

(1) Bounded and convergent (2) Convergent and unbounded

(3) Bounded and divergent (4) Divergent and unbounded

3. The radius of convergence of the series $1 - \frac{x}{1} + \frac{x^2}{2!} - \frac{x^3}{3!} + \ldots$ is _____

(1) ∞ (2) 0

(3) 1 (4) – 1

4. Which of the following continuous function f: $(0, \infty) \to \mathbf{R}$ can be extended to a continuous function on $[0, \infty)$?

(1) $f(x) = \sin \frac{1}{x}$ (2) $f(x) = \frac{1 - \cos x}{x^2}$

(3) $f(x) = \cos \frac{1}{x}$ (4) $f(x) = \frac{1}{x}$

5. A metric space X satisfies Bolzano-Weierstrass property of

(1) Every infinite sequence $< x_n >$ in X has no cluster point

(2) Every infinite sequence $< x_n >$ in X has at least one cluster point

(3) X is not sequentially compact

(4) X is not compact

6. If f is Riemann integrable with respect to α on [a, b], then

(1) f is increasing and α is a bounded function

(2) f is bounded and α is an increasing function

(3) f and α are both bounded

(4) f and α are both increasing function

SPACE FOR ROUGH WORK

7. The function $f(x) = \begin{cases} x \sin\frac{1}{x}, & x \neq 0 \\ 1, & x = 0 \end{cases}$ has

(1) Removal discontinuity at x = 0

(2) Non-removal discontinuity at x = 0

(3) Mixed discontinuity at x = 0

(4) Continuity at x = 0

8. Let f, g: [0, 1] → R be defined by

$f(x) = \begin{cases} x & if \quad x = \frac{1}{n} \ for \ n \in \mathbf{N} \\ 0 & otherwise \end{cases}$, and

$g(x) = \begin{cases} 1 & if \quad Qn\,[0,1] \\ 0 & otherwise \end{cases}$. Then

(1) Both f and g are Riemann integrable

(2) f is Riemann integrable and g is Lebesgue integrable

(3) g is Riemann integrable and f is Lebesgue integrable

(4) Neither f nor g is Riemann integrable

9. The set X = R with the metric $d(x, y) = \frac{|x - y|}{1 + |x - y|}$ is _____

(1) Bounded but not compact

(2) Bounded but not complete

(3) Complete but not bounded

(4) Compact but not complete

10. If $f(x, y) = \begin{cases} 3x + 4y, & (x, y) \neq (1, 2) \\ 6 & else \end{cases}$, then

$\lim_{(x,y) \to (e, \frac{1}{e})} f(x, y)$ is _____

(1) $e(3e^2 + 4)$

(2) $e^{-1}(3e^2 + 4)$

(3) $e(3 + 4e^2)$

(4) $e^{-1}(3 + 4e^2)$

SPACE FOR ROUGH WORK

11. Let f: $\mathbf{R}^2 \to \mathbf{R}$ be such that $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ exist at all points. Then

(1) The total derivative of f exists at all points of $\mathbf{R}^2$

(2) f is continuous on $\mathbf{R}^2$

(3) The function $f(x, y)$ as a function of x for every fixed y and $f(x, y)$ as a function of y for every fixed x are continuous

(4) All directional derivatives of f exist at all points of $\mathbf{R}^2$

12. Let $f(x) = x^2 + y^2 + 2x + 3y$. Then the directional derivatives of f(x) at (2, 1) in the direction of the vector $2i + j$ is _____

(1) $\frac{17}{5}$ (2) $\frac{17}{2}$

(3) $\frac{17}{\sqrt{5}}$ (4) $\frac{17}{\sqrt{2}}$

13. The characteristic function χ_A of a set A of finite measure is never

(1) A measurable function (2) A constant function

(3) An integrable function (4) A simple function

14. Let X and Y be normed linear spaces, and let $T : X \to Y$ be any bijective linear map with closed graph. Then, which one of the following statements is TRUE?

(1) The graph of T is equal to $X \times Y$ (2) T^{-1} is continuous

(3) The graph of T^{-1} is closed (4) T is continuous

15. Which of the following subsets of $\mathbf{R}^n$ is compact with respect to the usual topology of $\mathbf{R}^n$?

(1) $\{(x_1, x_2, \ldots, x_n) : |x_i| < 1, 1 \le i \le n\}$ (2) $\{(x_1, x_2, \ldots, x_n) : x_1 + x_2 + \ldots + x_n = 0\}$

(3) $\{(x_1, x_2, \ldots, x_n) : x_i \ge 0, 1 \le i \le n\}$ (4) $\{(x_1, x_2, \ldots, x_n) : 1 \le x_i \le 2^i, 1 \le i \le n\}$

16. Let V be the vector space of m x n matrices over a field F. Then the dimension of V is

(1) n (2) m

(3) mn (4) m – n

17. The eigen vectors of the matrix $\begin{bmatrix} 1 & 2 \\ 0 & 2 \end{bmatrix}$ are written in the form $\begin{bmatrix} 1 \\ a \end{bmatrix}$ and $\begin{bmatrix} 1 \\ b \end{bmatrix}$. What is a + b?

(1) 0 (2) 1/2

(3) 1 (4) 2

SPACE FOR ROUGH WORK

18. Let M be a 3 × 3 singular matrix and suppose that 2 and 3 are eigen values of M. Then the number of linearly independent eigenvectors of $M^3 + 2M + I_3$ is equal to _____

(1) 0 (2) 1

(3) 2 (4) 3

19. Let (1) a standard inner product on R^2. If $(\alpha/\gamma) = -1$ and $(\beta/\gamma) = 3$, given $\alpha = (1, 2)$ and $\beta = (-1, 1)$, then the value of γ is _____

(1) (0, 1) (2) (0, 3)

(3) $(0, \frac{2}{3})$ (4) (0, 0)

20. Let $\langle \bullet, \bullet \rangle : R^n \times R^n \to R$ be an inner product on the vector space R^n over R. Consider the following statements:

P: $|\langle u, v \rangle| \leq \frac{1}{2} (\langle u, u \rangle + \langle v, v \rangle)$ for all $u, v \in R^n$

Q: if $\langle u, v \rangle = \langle 2u, -v \rangle$ for all $v \in R^n$, then $u = 0$

Then

(1) Both P and Q are false (2) P is true and Q is false

(3) P is false and Q is true (4) Both P and Q are true

21. The symmetric matrix corresponding to the quadratic form $Q = x_1^2 + 2x_2^2 + 7x_3^2 + 2x_1x_3 + 6x_1x_3 + 10x_2x_3$ is _____

(1) $\begin{pmatrix} 1 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$ (2) $\begin{pmatrix} 1 & 1 & 3 \\ 1 & 2 & 5 \\ 3 & 5 & 7 \end{pmatrix}$

(3) $\begin{pmatrix} 1 & 0 & 1 \\ 3 & 2 & 5 \\ 1 & 7 & 1 \end{pmatrix}$ (4) $\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$

22. Let A, B be n × n real matrices such that det A > 0 and det B < 0. For $0 \leq t \leq 1$, consider C(t) = t A + (1 – t) B. Then

(1) C(t) is invertible for each $t \in [0, 1]$

(2) There is a $t_0 \in (0, 1)$ such that $C(t_0)$ is not invertible

(3) C(t) is not invertible for each $t \in [0, 1]$

(4) C(t) is invertible for only finitely many $t \in [0, 1]$

SPACE FOR ROUGH WORK

23. Let $A=\begin{bmatrix} 0 & 1 \\ -1 & 1 \end{bmatrix}$. Then the smallest positive integer n such that $A^n = I$ is

(1) 1 (2) 2

(3) 4 (4) 6

24. What is the rank of the quadratic form $Q(x) = 2x_1^2 + 2x_1x_2 + 2x_2^2 - 6x_1x_3 - 6x_2x_3 + 6x_3^2$?

(1) 2 (2) 3

(3) 4 (4) 5

25. Let $M=\begin{pmatrix} 1 & 1 & 2 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix}$ and $V = \{Mx^t : x \in R^3\}$. Then an orthonormal basis for V is

(1) $\left\{(1,0,0)^t, \left(0,\frac{2}{\sqrt{5}},\frac{1}{\sqrt{5}}\right)^t, \left(\frac{2}{\sqrt{6}},\frac{1}{\sqrt{6}},\frac{1}{\sqrt{6}}\right)^t\right\}$

(2) $\left\{(1,0,0)^t, \left(0,\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}\right)^t\right\}$

(3) $\left\{(1,0,0)^t, \left(\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}}\right)^t, \left(\frac{2}{\sqrt{6}},\frac{1}{\sqrt{6}},\frac{1}{\sqrt{6}}\right)^t\right\}$

(4) $\{(1,0,0)^t, (0,0,1)^t\}$

26. The critical points of $f(x) = x^3 + x^2 + x + 1$ are

(1) 2, 1 (2) – 2, – 1

(3) 2, – 1 (4) Do not exist

27. The angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at the point (2, – 1, 2) is

(1) $\cos^{-1}\left(\frac{8}{3\sqrt{21}}\right)$ (2) $\cos^{-1}\left(\frac{3}{8\sqrt{21}}\right)$

(3) $\sin^{-1}\left(\frac{8}{3\sqrt{21}}\right)$ (4) $\sin^{-1}\left(\frac{3}{8\sqrt{21}}\right)$

SPACE FOR ROUGH WORK

28. A vector field is given by F = sin y $\mathbf{I}$ + x($\mathbf{I}$ + cos y) J. The value of the line integral over a circular path given by $x^2 + y^2 = a^2$, z = 0 is

(1) πa^2 (2) $\pi^2 a$

(3) $\frac{\pi}{2} a^2$ (4) $\frac{\pi}{2} a$

29. If G is a group, for $x \in G$, N(x) is the normalizer of x, then for all $a \in N(x)$ is

(1) ax = xa (2) ax = e

(3) xa = e (4) $ax \neq xa$

30. The generators of a group $G = \{a, a^2, a^3, a^4, a^5, a^6 = e\}$ are

(1) a and a^5 (2) a^2 and a^4

(3) a^3 and a^5 (4) a^2 and a^3

31. Given permutation $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 1 & 2 & 5 & 4 \end{pmatrix}$ the matrix A is defined to be the one whose i^{th} column is the $\sigma(i)^{th}$ column of the identity matrix I. Which of the following is correct?

(1) $A = A^{-2}$ (2) $A = A^{-4}$

(3) $A = A^{-5}$ (4) $A = A^{-1}$

32. The remainder when 98! is divided by 101 is equal to

(1) 10 (2) 20

(3) 50 (4) 100

33. The number of 5-Sylow subgroups in a group of order 45 is

(1) 1 (2) 2

(3) 3 (4) 4

34. The element $a \neq 0 \in R$, the commutative ring is an integral domain if

(1) ab = 0, $b \in R$ and b = 0 (2) ab = 0, $b \in R$ and $b \neq 0$

(3) $ab \neq 0$, $b \in R$ and b = 0 (4) $ab \neq 0$, $b \in R$ and $b \neq 0$

SPACE FOR ROUGH WORK

35. If R is a commutative ring, with unit element then,

(1) Every maximal ideal is prime ideal

(2) Every prime ideal is maximal ideal

(3) Every ideal is prime ideal

(4) Every ideal is maximal ideal

36. Consider the quotient group Q/Z of the additive group of rational numbers. The order of the element $\frac{2}{3} + Z$ in Q/Z is

(1) 2 (2) 3

(3) 5 (4) 6

37. $\frac{Z_2(x)}{\langle (x^3 + x^2 + 1) \rangle}$ is

(1) a field having 8 elements (2) a field having 9 elements

(3) an infinite field (4) not a field

38. The function $f(z) = |z|^2 + i\bar{z} + 1$ is differentiable at

(1) i (2) 1

(3) – i (4) no point in $\mathbb{C}$

39. The maximum modulus of e^{z^2} on the set $S = \{z \in \mathbb{C} : 0 \leq Re(z) \leq 1, 0 \leq Im(z) \leq 1\}$ is

(1) 2/e (2) e

(3) e + 1 (4) e^2

40. The radius of convergence of the power series $\sum_{n=0}^{\infty} \frac{n!}{n^n} z^n$ is

(1) e (2) 0

(3) ∞ (4) 1/e

41. If a and b are the real numbers between 0 & 1 such that $z_1 = a + i$, $z_2 = 1 + ib$ and $z_3 = 0$ form an equilateral triangle, then a and b are

(1) $a = 2; b = 2 - \sqrt{3}$ (2) $a = 2 - \sqrt{3}; b = 2$

(3) $a = 2 + \sqrt{3}; b = 2 + \sqrt{3}$ (4) $a = 2 - \sqrt{3}; b = 2 - \sqrt{3}$

SPACE FOR ROUGH WORK

42. The power series $\sum_{n=0}^{\infty}\left(\frac{iz-1}{2+i}\right)^n$ is

(1) Convergent for $|z+i| < \sqrt{5}$

(2) Divergent for $|z+i| < \sqrt{5}$

(3) Absolutely convergent for $|z+i| = \sqrt{5}$

(4) Having infinite radius of convergence

43. Let $f: \mathbb{C} \to \mathbb{C}$ be defined by $f(x+iy) = x^3 + 3xy^2 + i(y^3 + 3x^2 y)$, $i = \sqrt{-1}$. Let $f'(z)$ be the derivative of f with respect to z, then which of the following is true?

(1) $f'(1+i)$ exists and $|f'(1+i)| = 3\sqrt{5}$

(2) f is analytic at the origin

(3) f is not differentiable at i

(4) f is differentiable at 1

44. Let $I = \int_C \frac{f(z)}{(z-1)(z-2)}\,dz$, where $f(z) = \sin\frac{\pi z}{2} + \cos\frac{\pi z}{2}$ and C is the curve $|z| = 3$ oriented anti-clockwise, then the value of I is

(1) $4\pi i$ (2) 0

(3) $-2\pi i$ (4) $-4\pi i$

45. If the function $f(z) = e^{-1/z^2}$, then which of the following is true?

(1) f (z) has infinitely many singularities

(2) f (z) has no singularities

(3) f (z) has an essential singularity at z = 0

(4) f (z) has a pole of order 2 at z = 0

46. The value of the integral $\int_C \frac{z(5z^3 - 6i)}{z^5 - 3iz^2 + i - 1}\,dz$ where C is the contour which encloses zero's of the function $f(z) = z^5 - 3iz^2 + i - 1$, is equal to

(1) $10\pi i$ (2) $5\pi i$

(3) $\frac{3}{5}\pi i$ (4) $\frac{5}{3}\pi i$

SPACE FOR ROUGH WORK

47. Let X be the real line in R. If A = (0, 1], then

(1) $\overline{A} = (0, 1]$

(2) $\overline{A} = (0, 1)$

(3) $\overline{A} = [0, 1]$

(4) $\overline{A} = [0, 1)$

48. Let X denote the product of countably many copies of [0, 1]. We let X_1 denote the set X equipped with the box topology and let X_2 denote the set X equipped with the product topology. Then

(1) X_1 is compact and separate

(2) X_2 is compact and separable

(3) X_1 and X_2 are both compact

(4) Neither X_1 nor X_2 is separable

49. Which of the following subsets of R^n is compact?

(1) $\{(x_1, x_2,....,x_n) : |x_i| < 1, 1 \le i \le n\}$

(2) $\{(x_1, x_2,....,x_n) : x_1 + x_2 + ...+ x_n = 0\}$

(3) $\{(x_1, x_2,....,x_n) : x_i \ge 0, 1 \le i \le n\}$

(4) $\{(x_1, x_2,....,x_n) : 1 \le x_i \le 2^i, 1 \le i \le n\}$

50. Let $X = \{a, b, c\}$, $T = \{\phi, \{a\}, \{a, b\}\}$ then

p) (X, T) is compact space

q) (X, T) is connected space

Then which of the following is true?

(1) Only p is true

(2) Only q is true

(3) Both p and q are ture

(4) Both p and q are not true

51. For which value of λ all the solutions of $x^2\left(\frac{d^2y}{dx^2}\right) - 3x\left(\frac{dy}{dx}\right) - \lambda y = 0$ tends to zero as $x \to \infty$

(1) $2 \le \lambda < 1$

(2) $-1 \ge \lambda > 0$

(3) $-1 \le \lambda < 0$

(4) $1 \le \lambda \le 2$

SPACE FOR ROUGH WORK

52. The Wronskian of two solutions y_1, y_2 of the differential equation $\frac{d^2y}{dx^2}+x\frac{dy}{dx}+xy=0$ is

(1) W (y_1, y_2; x) = $ke^{-x^2/2}$ where k is constant

(2) W (y_1, y_2; x) = $ke^{x^3/2}$ where k is constant

(3) W (y_1, y_2; x) = $k\left(\frac{x^2}{2}\right)$ where k is constant

(4) W (y_1, y_2; x) = $ke^{-x/2}$ where k is constant

53. The maximum number of linearly independent solutions of the differential equation $\frac{d^4y}{dx^4}=0$ with the condition y (0) = 1 is

(1) 4 (2) 3

(3) 2 (4) 1

54. The solution of the differential equation $y'' = f(x)\ \forall\ x\in(0, 1)$, y(0) = 0, y(1) = 0 is given by

$y(x)=\int_0^1 G(x, s)\, f(s)\, ds$ where _____

(1) $G(x,s)=\begin{cases}x(s-1) & x\le s\\ \xi(x-1) & x>s\end{cases}$ (2) $G(x,s)=\begin{cases}x^2(s-1) & x\le s\\ s^2(x-1) & x<s\end{cases}$

(3) $G(x,s)=\begin{cases}x(s^2-1) & x\le s\\ s(x^2-1) & x>\xi\end{cases}$ (4) $G(x,s)=\begin{cases}\sin x(s-1) & x\le s\\ \sin s(x-1) & x>s\end{cases}$

55. Let u (x, y) be the solution of the Cauchy problem $\frac{\partial u}{\partial y}-x\frac{\partial u}{\partial x}+u-1=0$.

Where $-\infty<x<\infty, y\ge 0$ and $u(x, 0) = \sin x$. Then $u(0, 1)$ is equal to

(1) 1 – 1/e (2) 1 + 1/e

(3) $1-\frac{1-\sin e}{e}$ (4) $1+\frac{1-\sin e}{e}$

56. The equation of the integral surface of the differential equation $(x^2 - yz)\,p + (y^2 - zx)\,q = z^2 - xy$

Where $p=\frac{\partial z}{\partial x}$ and $q=\frac{\partial z}{\partial y}$, which passes through the line x = 1, y = 0 is

(1) (xy + yz + zx) (x – y) = y (2) (x – y) (xy + yz + zx) + y – z = 0

(3) (x + y) (xy + yz + zx) + y – z = 0 (4) (x + y) (xz + yx – zy) + x – y = 0

SPACE FOR ROUGH WORK

57. With regard to the first order partial differential equations with p and q having their usual meaning. Which of the following statements is NOT TRUE?

(1) $y x^2 p + x y^2 q = x y z + x^2 y^3$ is linear

(2) $p + q = z + x y$ is nonlinear

(3) $x y p + x^2 y q = x^2 y^2 z^2$ is semilinear

(4) $p^2 + q^2 = 1$ is nonlinear

58. The second order partial differential equation $\frac{\partial^2 u}{\partial x^2} + 4\frac{\partial^2 u}{\partial x \partial y} - 2\frac{\partial^2 u}{\partial y \partial z} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0$ is of the type

(1) Parabolic

(2) Hyperbolic

(3) Nonlinear

(4) Elliptic

59. The system of *m*-linear algebraic equations with *n*-unknowns will have no solution when

(1) $m = n$

(2) $m < n$

(3) $m > n$

(4) $m \geq n$

60. The rate of convergence of the Newton-Raphson method is

(1) 1

(2) 1.62

(3) 3

(4) 2

61. If the fourth order Runge-Kutta method is used to solve the initial value problem $\frac{du}{dt} = u$, $u(o) = \alpha$ and obtained $u(1) = 1$ with a step size of $h = 1$, then the value of k_4 is

(1) 2.14

(2) 1.017

(3) 2.65

(4) 1.67

62. The quadratic spline in [0,1] with $M(0) = f''(0) = 0$ which fits the data

x	0	1	2	3
f(x)	1	2	33	244

is

(1) $P(x) = 2x^2 + 3$

(2) $P(x) = x - 1$

(3) $P(x) = x + 1$

(4) $P(x) = 2x^3 + 1$

SPACE FOR ROUGH WORK

63. Let I [y (x)] = $\int_0^1 (1+y''^2)\,dx$ satisfying y (0) = 0, y′(0) = 1, y(1) = y′(1) = 1 where unknown y (x) possesses fourth order derivative everywhere in (0, 1). Then the Euler equation for extremizing the functional I [y (x)] is of _____

(1) Second order linear
(2) Fourth order linear
(3) Fifth order nonlinear
(4) Third order nonlinear

64. Suppose that the function

F (x, y, y′) = y′ – y² + B x y be such that the equation y″ + y = –x,

y (0) = y (1) = 0 is Euler's equation for the extremum of the functional

I [y (x)] = $\int_0^1 F(x, y, y')\,dx$, then the value of B is

(1) –1
(2) +2
(3) 0
(4) –2

65. The extremal of the functional I [y(x)] = $\int_0^1 \frac{y'^2}{x^3}\,dx$ satisfying the boundary conditions y (0) = 0 and y (1) = 1 is

(1) $y = x^2$
(2) $y = x^3$
(3) $y = x^4$
(4) $y = x$

66. Laplace transform of $t^2\cos 2t$ is

(1) $\frac{6s^2-2}{(s^2+1)^3}$
(2) $\frac{s^2-6}{(s+1)^3}$
(3) $\frac{6s^2+2}{(s^2-1)^3}$
(4) $\frac{6s^2+1}{(s^2+2)^3}$

67. Laplace transform of $\frac{\sin^2 t}{t}$ is

(1) $\log\frac{\sqrt{s^2+4}}{s}$
(2) $\frac{3}{2}\log\frac{\sqrt{s+1}}{s}$
(3) $\frac{1}{2}\log\frac{\sqrt{s^2+4}}{s}$
(4) $\frac{1}{2}\log\frac{\sqrt{s^2-4}}{s}$

SPACE FOR ROUGH WORK

68. If $L\{f(t)\} = \frac{5s+4}{s^2-g}$ then $\int_0^\infty e^{-2t} f(t)\,dt$ is

(1) $\frac{-14}{5}$ (2) $\frac{5}{14}$

(3) $\frac{-7}{15}$ (4) $\frac{6}{13}$

69. The inverse Laplace transform of $\frac{s}{(s+3)^2}$ is

(1) $e^{3t} + 3te^{-3t}$ (2) $e^{t} - 3te^{-t}$

(3) $e^{-3t} - 3te^{3t}$ (4) $e^{-3t} - 3te^{-3t}$

70. The inverse Laplace transform of $\frac{1}{S(S^2+4)}$ using convolution theorem is

(1) $\frac{1}{2}(1 + \cos 2t)$ (2) $\frac{1}{4}(1 + \cos 2t)$

(3) $\frac{1}{4}(1 - \cos 2t)$ (4) $\frac{1}{2}(1 + \cos 4t)$

71. Using finite Fourier transform, the solution of $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$, given $u(0, t) = 0$, $u(4, t) = 0$ and $u(x, 0) = 2x$, where $0 < x < 4$, $t > 0$ is

(1) $16\sum_{n=1}^{\infty} \frac{(-1)^n}{n\pi} e^{-n^2\pi^2 t/16} \sin\left(\frac{n\pi x}{4}\right)$ (2) $16\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n\pi} e^{-n^2\pi^2 t/16} \sin\left(\frac{n\pi x}{4}\right)$

(3) $16\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n\pi} e^{-n^2\pi^2 t/16} \cos\left(\frac{n\pi x}{4}\right)$ (4) $16\sum_{n=1}^{\infty} \frac{(-1)^{n}}{n\pi} e^{-n^2\pi^2 t/16} \cos\left(\frac{n\pi x}{4}\right)$

72. The sine and cosine Fourier transforms of x^{n-1}, $n > 0$, respectively are

(1) $F_s(x^{n-1}) = \frac{\Gamma(n)}{s^n} \sin\frac{n\pi}{2}$; $F_c(x^{n-1}) = \frac{\Gamma(n)}{s^n} \cos\frac{n\pi}{2}$

(2) $F_s(x^{n-1}) = \frac{\Gamma(n)}{s^n} \cos\frac{n\pi}{2}$; $F_c(x^{n-1}) = \frac{\Gamma(n)}{s^n} \sin\frac{n\pi}{2}$

(3) $F_s(x^{n-1}) = \frac{\Gamma(n)}{s^n} \cos\frac{\pi}{2n}$; $F_c(x^{n-1}) = \frac{\Gamma(n)}{s^n} \sin\frac{\pi}{2n}$

(4) $F_s(x^{n-1}) = \frac{\Gamma(n)}{s^n} \sin\frac{\pi}{2n}$; $F_c(x^{n-1}) = \frac{\Gamma(n)}{s^n} \cos\frac{\pi}{2n}$

SPACE FOR ROUGH WORK

73. An infinite string is initially at rest, and the initial displacement is f(x), $(-\infty < x < \infty)$. Then the displacement y(x, t) of the string is

(1) $\frac{1}{2}$ [f(x – ct) – f(x + ct)]

(2) $\frac{1}{2}$ [f(x – ct) + f(x + ct)]

(3) $\frac{1}{4}$ [f(x – ct) – f(x + ct)]

(4) $\frac{1}{4}$ [f(x – ct) + f(x + ct)]

74. A bar of length 'a' is at temperature zero. At t = 0, the end x = a is suddenly raised to temperature u_0 and the end x = 0 is insulated. The temperature u(x, t) at any point x of the bar at any time t > 0, assuming that the surface of the bar is insulated is

(1) $u_0 + \frac{4u_0}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^n}{2n-1}e^{-(2n-1)^2\pi^2c^2t/4a^2}\cdot\cos\left\{\frac{(2n-1)\pi x}{2a}\right\}$

(2) $u_0 + \frac{4u_0}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^n}{2n+1}e^{-(2n+1)^2\pi^2c^2t/4a^2}\cdot\cos\left\{\frac{(2n+1)\pi x}{2a}\right\}$

(3) $u_0 + \frac{2u_0}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^n}{2n-1}e^{-(2n-1)^2\pi^2c^2t/4a^2}\cdot\cos\left\{\frac{(2n-1)\pi x}{2a}\right\}$

(4) $u_0 + \frac{2u_0}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^n}{(2n+1)}e^{-(2n+1)^2\pi^2c^2t/4a^2}\cdot\cos\left\{\frac{(2n+1)\pi x}{2a}\right\}$

75. If the Fourier sine transform of f(x) is $\frac{1-\cos n\pi}{n^2\pi^2}$; $0 \le x \le \pi$, then the value of f(x) is

(1) $\frac{\pi^3}{2}\sum_{n=1}^{\infty}\left\{\frac{1-\sin n\pi}{n^2}\right\}\cos nx$

(2) $\frac{2}{\pi^3}\sum_{n=1}^{\infty}\left\{\frac{1-\sin n\pi}{n^2}\right\}\cos nx$

(3) $\frac{2}{\pi^3}\sum_{n=1}^{\infty}\left\{\frac{1-\cos n\pi}{n^2}\right\}\sin nx$

(4) $\frac{\pi^3}{2}\sum_{n=1}^{\infty}\left\{\frac{1-\sin n\pi}{n^2}\right\}\cos nx$

SPACE FOR ROUGH WORK

76. In a central orbit if v is the linear velocity at any point of the path, then

(1) $v^2 = h^2\left[u^2 + \frac{d^2u}{d\theta^2}\right]$

(2) $v^2 = h^2\left[u^2 + \left(\frac{du}{d\theta}\right)^2\right]$

(3) $v^2 = h^2\left[u + \frac{du}{d\theta}\right]$

(4) $v^2 = h^2\left[u^2 + \frac{d^2\psi}{d\theta^2}\right]$

77. A force $5\hat{i} - 2\hat{j} + 3\hat{k}$ acts on a particle with position vector $2\hat{i} + \hat{j} - 2\hat{k}$. The torque of the force about the origin is

(1) $\hat{i} + 16\hat{j} + 9\hat{k}$

(2) $-\hat{i} - 16\hat{j} - 9\hat{k}$

(3) $\hat{i} + 16\hat{j} - 9\hat{k}$

(4) $\hat{i} - 16\hat{j} + 9\hat{k}$

78. If the Hamiltonian of a dynamical system is given by $H = pq - q^2$, then as $t \to \infty$

(1) $q \to \infty$, $p \to \infty$

(2) $q \to 0$, $p \to 0$

(3) $q \to \infty$, $p \to 0$

(4) $q \to 0$, $p \to \infty$

79. If two masses, M_1 and M_2 are connected by an inextensible string of length L, and if X be the variable vertical distance from the pulley to the mass M_1 and mass M_2 will be at a distance (L – X) from the pulley, then which of the following is NOT true?

(1) $\ddot{X} = \frac{M_1 - M_2}{M_1 + M_2} g$ is Lagrangian equation of motion

(2) $-M_1 g X - M_2 g(L - X)$ is the potential energy of the system

(3) $\frac{1}{2}(M_1 + M_2)\dot{X}^2$ is the kinetic energy of the system

(4) $\ddot{X} = (M_1 - M_2) g$ is the Lagrangian equation of motion

80. Which exploratory data analysis technique is used to visualize the relationship between two continuous variables with a line connecting the data points?

(1) Scatter plot

(2) Histogram

(3) Bar chart

(4) Box plot

SPACE FOR ROUGH WORK

81. For a negatively skewed distribution, the correct relation between mean, median and mode is

(1) Mean = Median = Mode
(2) Median < Mean < Mode
(3) Mean < Median < Mode
(4) Mode < Mean < Median

82. Let X and Y have the joint probability mass function

$$P(X = m, Y = n) = \begin{cases} \frac{m+n}{21}, & if \ m = 1,2,3 \\ & n = 1,2 \\ 0 & o.w. \end{cases}$$

Then P(X = 2/Y = 2) equals

(1) 1/3
(2) 2/3
(3) 1/2
(4) 1/4

83. A company has two plants to manufacture televisions. Plant I manufactures 70% of televisions and Plant II manufactures 30%. At plant I, 80% of the televisions are rated to be of standard quality and at plant II 90% of the televisions are rated to be of standard quality. A television is chosen at random and is found to be of standard quality. The probability that it has come from plant II is

(1) $\frac{17}{50}$
(2) $\frac{27}{83}$
(3) $\frac{3}{5}$
(4) $\frac{4}{50}$

84. A man is known to speak truth 2 out of 3 times. He throws a die and reports that the number obtained is a four. Find the probability that the number obtained is actually a four.

(1) $\frac{1}{14}$
(2) $\frac{2}{7}$
(3) $\frac{3}{7}$
(4) $\frac{5}{7}$

85. Suppose the expected runtime of Quicksort in $2n \log(n)$ operations, to sort an array of size n. What is the upper bound in the sense of Markov's inequality for the probability that Quicksort runs for longer then $20n \log(n)$ time?

(1) 9%
(2) 10%
(3) 11%
(4) 12%

SPACE FOR ROUGH WORK

86. Consider a Markov chain $X_0, X_1, X_2 \ldots$ with state space S. Suppose i, j $\in$ S are two states which communicate with each other. Which of the following statements is not correct?

(1) Period of i = period of j

(2) i is recurrent if and only if j is recurrent

(3) $\lim_{n\to\infty} P[x_n = i \mid x_0 = k] = \lim_{n\to\infty} P[x_n = j \mid x_0 = k]$ for all k $\in$ S

(4) $\lim_{n\to\infty} P[x_n = j \mid x_0 = i] = \lim_{n\to\infty} P[x_n = j \mid x_0 = j]$

87. A firm produces three products which are processed on three machines. The relevant data is given below:

Machine	Time per unit (minutes)			Machine Capacity (minutes/day)
	Product A	Product B	Product C	
M_1	2	3	2	440
M_2	4	–	3	470
M_3	2	5	–	430

The profit per unit of products A, B and C is ₹ 4, ₹ 3, ₹ 6 respectively. What is the daily number of units to be manufactured for each product, so that the profit is maximized? Assume that all the units produced are consumed in the market.

(1) $x_1 = 0, x_2 = \frac{380}{9}, x_3 = \frac{470}{3}$

(2) $x_1 = 0, x_2 = 20, x_3 = 200$

(3) $x_1 = 20, x_2 = 40, x_3 = 60$

(4) $x_1 = \frac{380}{9}, x_2 = 0, x_3 = \frac{470}{3}$

SPACE FOR ROUGH WORK

88. A company has 3 cement factories located in cities 1, 2, 3 which supply cement to 4 projects located at towns 1, 2, 3, 4. Each plant can supply 6, 1, 10 truck loads of cement daily, respectively and the daily cement requirements of the projects are 7, 5, 3, 2 truck loads respectively. The transportation costs per truck load in hundreds of rupees from each plant to each project site are provided below:

Factories \ Projects	1	2	3	4
1	2	3	11	7
2	1	0	6	1
3	5	8	15	9

What is the optimal transportation cost?

(1) ₹ 8,000/-

(2) ₹ 9,000/-

(3) ₹ 10,000/-

(4) ₹ 11,000/-

89. In the Hungarian method of solving an assignment problem, the row reduction is obtained by

(1) Dividing each row by the elements of the row above it

(2) Subtracting the elements of the row from the elements of the row above it

(3) Subtracting the smallest element from all other elements of the row

(4) Subtracting all the elements of the row from the highest elements in the matrix

90. The size of the payoff matrix of a game can be reduced by using the principal of

(1) game inversion

(2) rotation reduction

(3) dominance

(4) game transpose

SPACE FOR ROUGH WORK

91. Find the total cost using Vogel's approximation method.

Industries	Centres				Supply
	C_1	C_2	C_3	C_4	
I_1	3	2	7	6	50
I_2	7	5	2	3	60
I_3	2	5	4	5	25
Demand	60	40	20	15	

(1) 400 (2) 420

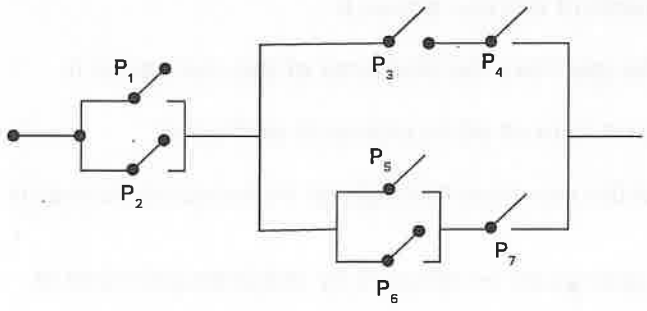
(3) 440 (4) 460

92. The set A = {1, 2, 4, 5, 10, 20} if they exist in the poset $(z^+, /)$, the greatest lower bound and least upper bound are

(1) (1, 20) (2) (1, 10)

(3) (2, 20) (4) (2, 10)

93. What is the complement of the Boolean function represented by the following circuit?



(1) $(P_1 \vee P_2)' \vee [(P_3 \wedge P_4) \wedge ((P_5 \wedge P_6) \vee P_7)]$

(2) $(P_1 \vee P_2) \wedge [(P_3 \wedge P_4) \vee ((P_5 \vee P_6) \wedge P_7)]$

(3) $(P_1' \vee P_2') \wedge [(P_3 \wedge P_4) \vee ((P_5 \vee P_6) \wedge P_7)]$

(4) $(P_1' \wedge P_2') \vee [(P_3' \vee P_4') \wedge ((P_5' \wedge P_6') \vee P_7')]$

SPACE FOR ROUGH WORK

94. Let $y_1(x) = x^3$ and $y_2(x) = x^2|x|$ for $x \in \mathbf{R}$, consider the following statements

P : $y_1(x)$ and $y_2(x)$ are linearly independent solutions of $x^2\left(\frac{d^2y}{dx^2}\right) - 4x\left(\frac{dy}{dx}\right) + 6y = 0$ on $\mathbf{R}$.

Q: The Wronskian of y_1 and y_2 is zero $\forall x \in \mathbf{R}$

Which of the following statements is true?

(1) Both P and Q
(2) Only P
(3) Only Q
(4) Neither P nor Q

95. Let T be a tree with 40 edges. The removal of certain edge from T yields 2 disjoint Trees T_1 and T_2. Given that the number of vertices in T_1 equals the number of edges in T_2, determine the number of vertices and number of edges in T_1 and T_2.

(1) (22, 20)
(2) (21, 20)
(3) (20, 40)
(4) none of the above

96. Which of the following statement is INCORRECT?

(1) The number of vertices in a binary tree is odd
(2) $K(G) \leq \lambda(G) \leq \delta(G)$
(3) A graph G has an Eulerian cycle if it contains atmost 2 vertices of odd degree
(4) Petersen graph is neither Eulerian nor Hamiltonian

97. Traveling Salesman Problem is as followed:

From \ To	A	B	C	D
A	–	46	16	40
B	41	–	50	40
C	82	32	–	60
D	40	40	36	–

Optimal assignment schedule is

(1) $A \to B \to C \to D \to A$
(2) $A \to C \to B \to D \to A$
(3) $A \to B \to D \to C \to A$
(4) $A \to D \to C \to B \to A$

SPACE FOR ROUGH WORK

98. The adjacency matrix A of a tripartite graph whose two parts have m and n vertices can be written in the form

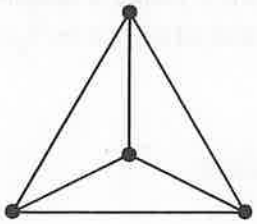
(1) $A = \begin{pmatrix} 0_{m,m} & B \\ B^T & 0_{n,n} \end{pmatrix}$

(2) $A = \begin{pmatrix} B & 0_{m,m} \\ 0_{n,n} & B^T \end{pmatrix}$

(3) $A = \begin{pmatrix} 0_{m,m} & 0_{n,n} \\ B & B^T \end{pmatrix}$

(4) $A = \begin{pmatrix} 0_{m,m} & B \\ 0_{n,n} & B^T \end{pmatrix}$

99. For the graph given below which of the statements is correct?



(1) Graph is Eulerian, but not Hamiltonian

(2) Graph is both Eulerian and Hamiltonian

(3) Graph is Hamiltonian but not Eulerian

(4) Graph is neither Hamiltonian nor Eulerian

100. The maximum value of $z = 2x + 3y$ subject to the constraints

$x + y \leq 30$, $y \geq 3$, $0 \leq y \geq 12$,

$x - y \geq 0$, $0 \leq x \leq 20$ is

(1) 72

(2) 73

(3) 74

(4) 75

SPACE FOR ROUGH WORK