DO NOT OPEN THIS TEST BOOKLET UNTIL YOU ARE ASKED TO DO SO

TEST BOOKLET

Sl. No. 0434

Subject Code : 18

Subject : Mathematics

LECTURERS FOR NON-GOVT. AIDED COLLEGES OF ODISHA

Time Allowed : 2 Hours

Maximum Marks : 150

: INSTRUCTIONS TO CANDIDATES :

1. **IMMEDIATELY AFTER THE COMMENCEMENT OF THE EXAMINATION, YOU SHOULD CHECK THAT THIS TEST BOOKLET CONTAINS 23 PAGES AND DOES NOT HAVE ANY UNPRINTED OR TORN OR MISSING PAGES OR ITEMS ETC. IF SO, GET IT REPLACED BY A COMPLETE TEST BOOKLET.**

2. You have to enter your **Roll No.** on the Test Booklet in the Box provided alongside. **DO NOT** write anything else on the Test Booklet.

3. The Test Booklet contains **100** questions. Each question comprises four answers. You have to select the correct answer which you want to mark (darken) on the **Answer Sheet (OMR Sheet)**. In any case choose ONLY ONE answer for each question. If more than one answer is darkened, it will be considered as wrong.

4. You have to mark (darken) all your answers only on the **OMR Answer Sheet using BLACK BALL POINT PEN** provided by the State Selection Board. You have to do rough work only in the space provided at the end of the Test Booklet. See instructions in the Answer Sheet.

5. All questions carry equal marks i.e. of one and half mark for each correct answer and each wrong answer will result in negative marking of **0.50** mark.

6. Before you proceed to mark (darken) the answers in the **OMR Answer Sheet** to the questions in the Test Booklet, you have to fill in some particulars in the Answer Sheet as per the instructions in your Admit Card.

7. On completion of the examination, you should hand over the **original Answer Sheet (OMR Sheet)** issued to you to the Invigilator before leaving the Examination Hall. You are allowed to take with you the candidate's copy (carbon copy) of the **OMR Answer Sheet** along with the Test Booklet for your reference.

SEAL

Candidate's full signature

Invigilator's signature

1. A non-empty subset H of a group G is a subgroup of G if $a \in H, b \in H \Rightarrow ab \in H$ and :

(A) $a \in H, b \in H \Rightarrow ab = ba$

(B) $(ab)c = a(bc)$ for all $a, b, c \in H$

(C) there exists an identity element $e \in G$ such that $ea = ae = a$ for all $a \in H$

(D) $a \in H \Rightarrow a^{-1} \in H$ where a^{-1} is the inverse of a in G

2. The subgroup N of a group G is a normal subgroup of G if and only if :

(A) It has only left coset of N in G

(B) It has only right coset of N in G

(C) Every left coset of N in G is a right coset of N in G

(D) $gN \neq Ng$ for every $g \in G$

3. A mapping ϕ from a group G into a group $\bar{G}$ is said to be a homomorphism if for all $a, b \in G$:

(A) $\phi(ab) = \phi(a) + \phi(b)$

(B) $\phi(ab) = \phi(a) - \phi(b)$

(C) $\phi(ab) = \phi(a)\,\phi(b)$

(D) $\phi(ab) = \phi(a) / \phi(b)$

4. A homomorphism ϕ from G into $\bar{G}$ is said to be an isomorphism if :

(A) ϕ is onto

(B) ϕ is into

(C) ϕ is one to one

(D) ϕ is many to one

5. A field is a :

(A) Division ring

(B) Commutative ring

(C) Commutative division ring

(D) Intergral domain

6. If ϕ is a homomorphism from a ring R into the ring R′, then :

(A) $\phi(-a) = \phi(a)$

(B) $\phi(-a) = -\phi(a)$

(C) $\phi(-a) = [\phi(a)]^{-1}$

(D) $\phi(-a) = \phi(a^{-1})$

7. If U is an ideal of the ring R then :

(A) R + U is a ring

(B) R/U is a ring

(C) RU is a ring

(D) R – U is a ring

8. The g. c. d. of 12 and 30 is :

(A) 4

(B) 5

(C) 6

(D) 7

9. For what value of p, 3p + 1 is a perfect square ?

(A) 17

(B) 13

(C) 7

(D) 5

10. If g. c. d. of (a, 30) = 1 then $a^4 + 59$ is divisible by :

(A) 60

(B) 61

(C) 62

(D) 63

11. A homomorphism of a group into itself is called :

(A) An isomorphism

(B) An endomorphism

(C) An automorphism

(D) An abelian

12. The product of any four consecutive integers is divisible by :

(A) 22

(B) 24

(C) 25

(D) 26

13. If S and T are subsets of a vector space V and if $S \subset T$ then :

(A) $L(S) \subset L(T)$

(B) $L(T) \subset L(S)$

(C) $L(T) = L(S)$

(D) $L(S \cup T) = L(S) - L(T)$

14. If W is a proper subspace of a finite-dimensional vector space V then W is finite dimensional and :

(A) dim V < dim W

(B) dim W < dim V

(C) dim V = dim W

(D) dim W ≤ dim V

15. If V be the vector space of all 2 × 2 matrices over the field F, then dimension of V is :

(A) 3

(B) 4

(C) 5

(D) 6

16. If $T : V_4 \rightarrow V_3$ be a linear transformation defined by $T(x_1, x_2, x_3, x_4) = (x_1 - x_4, x_2 + x_3, x_3 - x_4)$ then its rank is :

(A) 2

(B) 3

(C) 4

(D) 5

17. If $T : U \rightarrow V$ be a linear map and if U is finite dimensional then :

(A) dim U = dim R(T)

(B) dim U < dim R(T)

(C) dim R(T) < dim U

(D) dim R(T) ≤ dim U

18. The nullity of the matrix $\begin{bmatrix} 1 & -1 & 2 \\ 3 & -2 & 5 \end{bmatrix}$ is :

(A) 0

(B) 1

(C) 2

(D) 3

19. The eigen values of the matrix $\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 0 \end{bmatrix}$ are :

(A) 2, 3, 4

(B) 2, 1, 0

(C) 1, 2, 3

(D) 1, 1, 0

20. If A is an m × n matrix and B is an n × p matrix then $(AB)^T$ is :

(A) $A^T B^T$

(B) $B^T A^T$

(C) $(BA)^T$

(D) $B^T A$

21. The union of an arbitrary family of open set is :

(A) closed

(B) open

(C) closed and open

(D) neither closed nor open

22. If S and T are subsets of real numbers then $(S \cup T)'$ is :

(A) $S' \cap T'$

(B) $S \cup T'$

(C) $S' \cup T'$

(D) $S \cap T$

23. The set R of real numbers is :

(A) open

(B) closed

(C) open as well as closed

(D) neither open nor closed

24. The set of all order pairs of positive integers is :

(A) uncountable

(B) countable

(C) finite

(D) uncountable and finite

25. A necessary and sufficient condition for the convergence of a sequence is that it is bounded and :

(A) has no limit point

(B) has finite number of limit points

(C) has a unique limit point

(D) has infinite number of limit points

26. The sequence $\{b_n\}$ where $b_n =$

$$\left\{\frac{1}{\sqrt{n^2+1}}+\frac{1}{\sqrt{n^2+2}}+.....+\frac{1}{\sqrt{n^2+n}}\right\}$$

converges to :

(A) 4

(B) 3

(C) 2

(D) 1

27. If $f(x) = \begin{cases} 2x-3 & \text{when } 0 \le x \le 2 \\ x^2-3 & \text{when } 2 < x \le 4 \end{cases}$

then at $x = 2$, the function is :

(A) Not derivable

(B) Derivable

(C) Not continuous

(D) Derivable and continuous

28. The function $f(x) = (x-3)^5 (x+1)^4$ has :

(A) maxima at $x = 3$

(B) minima at $x = 3$

(C) neither maxima nor minima at $x = 3$

(D) has point of inflection at $x = 3$

29. In the interval $]0, 1[$, the function $f(x) = \frac{1}{x}$ is :

(A) not uniformly continuous

(B) uniformly continuous

(C) continuous

(D) not continuous

30. In any interval containing zero, the sequence $\{f_n\}$ where $f_n(x) = \frac{nx}{1+n^2x^2}$ is :

(A) uniformly convergence

(B) not uniformly convergence

(C) convergence

(D) not convergence

31. The limit of $\lim_{n\to\infty} \frac{1}{n}\left[\left(\frac{1}{n}\right)^2 + \left(\frac{2}{n}\right)^2 + \cdots + \left(\frac{n}{n}\right)^2\right]$ is :

(A) $\frac{1}{2}$

(B) $\frac{1}{3}$

(C) $\frac{1}{4}$

(D) $\frac{1}{5}$

32. If the set $E \subset [a, b]$ is measurable and if c and d are in (a, b) such that $c < d$ then [c, d] is :

(A) not measurable

(B) measurable

(C) only outer measure exists

(D) only inner measure exists

33. For any two partitions P_1, P_2 :

(A) $L(P_1, f) \geq U(P_2, f)$

(B) $L(P_1, f) < U(P_2, f)$

(C) $L(P_1, f) \leq U(P_2, f)$

(D) $L(P_1, f) > U(P_2, f)$

34. Every finite subset of any metric space is :

(A) not bounded

(B) not closed

(C) not compact

(D) compact

35. Every bounded infinite subset of R has :

(A) no limit point

(B) only one limit point

(C) at least one limit point

(D) two limit points

36. A subset S of the set of real numbers R is compact if :

(A) S is closed

(B) S is bounded

(C) S is closed and bounded

(D) S is not closed and not bounded

37. A bounded function f having a finite number of points of discontinuity on [a, b] is :

(A) not differentiable on [a, b]

(B) differentiable on [a, b]

(C) not integrable on [a, b]

(D) integrable on [a, b]

38. If f is a non-negative continuous function on [a, b] and $\int_a^b f dx = 0$, then for all $x \in [a, b]$:

(A) $f(x) = 0$

(B) $f(x) = \dot{2}$

(C) $f(x) = 3$

(D) $f(x) = 4$

39. A series of function $\sum f_n$ converges uniformly on [a, b] if there exists a convergent series $\sum M_n$ of positive numbers such that for all $x \in [a, b]$:

(A) $M_n \leq |f_n(x)|$ for all n

(B) $|f_n(x)| \leq M_n$ for all n

(C) $|f_n(x)| < M_n$ for all n

(D) $M_n < |f_n(x)|$ for all n

40. If $-1 < x < 1$, then $\frac{1}{1+x}+\frac{2x}{1+x^2}+\frac{4x^3}{1+x^4}+\cdots\cdots$ is :

(A) $\frac{1}{1+x}$

(B) $\frac{1}{1-x}$

(C) $\frac{2}{1+x}$

(D) $\frac{2}{1-x}$

41. The central difference operator δ is defined in terms of shift operator E as :

(A) $E^{\frac{1}{2}}+E^{-\frac{1}{2}}$

(B) $E^{-\frac{1}{2}}-E^{\frac{1}{2}}$

(C) $E^{\frac{1}{2}}-E^{-\frac{1}{2}}$

(D) $\frac{1}{2}\left(E^{-\frac{1}{2}}+E^{\frac{1}{2}}\right)$

42. The averaging operator μ is defined in terms of shift operator E as :

(A) $E^{\frac{1}{2}}+E^{-\frac{1}{2}}$

(B) $\frac{1}{2}\left(E^{\frac{1}{2}}+E^{-\frac{1}{2}}\right)$

(C) $\frac{1}{2}\left(E^{\frac{1}{2}}-E^{-\frac{1}{2}}\right)$

(D) $E^{\frac{1}{2}}-E^{-\frac{1}{2}}$

43. In Secant method, the (n+1)th approximation x_{n+1} is written as :

(A) $x_n+\frac{x_n-x_{n-1}}{f(x_n)+f(x_{n-1})}f(x_n)$

(B) $x_n-\frac{x_n+x_{n-1}}{f(x_n)-f(x_{n-1})}f(x_n)$

(C) $x_n+\frac{x_n-x_{n-1}}{f(x_n)-f(x_{n-1})}f(x_n)$

(D) $x_n-\frac{x_n-x_{n-1}}{f(x_n)-f(x_{n-1})}f(x_n)$

44. If E, μ and δ are shift operator, averaging operator and central difference operator respectively then $E^{\frac{1}{2}}$ can be written as :

(A) $\mu-\frac{1}{2}\delta$

(B) $\mu+\frac{1}{2}\delta$

(C) $\delta+\frac{1}{2}\mu$

(D) $\delta-\frac{1}{2}\mu$

45. In the differential equation $(D^2 - 2D + 4)y = e^x \cos x$, the particular integral is :

(A) $2e^x \cos x$

(B) $e^x \cos x$

(C) $\frac{1}{2}e^x \cos x$

(D) $\frac{1}{3}e^x \cos x$

46. In the differential equation $(x^2D^2 + xD + 1)y = \log x \sin(\log x)$, the particular integral is :

(A) $\frac{1}{4}(\log x)^2 \cos(\log x) + \frac{1}{4} \log (\log x)$

(B) $-\frac{1}{4}(\log x)^2 \cos(\log x)$

(C) $\frac{1}{4} \log (\log x) \sin(\log x)$

(D) $-\frac{1}{4}(\log x)^2 \cos(\log x) + \frac{1}{4} \log (\log x) \sin(\log x)$

47. Laplace transform of $e^{at} \sinh bt$ is :

(A) $\frac{s+a}{(s+a)^2+b^2}$

(B) $\frac{s-a}{(s-a)^2-b^2}$

(C) $\frac{s+a}{(s+a)^2-b^2}$

(D) $\frac{s-a}{(s-a)^2+b^2}$

48. If $L\left(\frac{\sin t}{t}\right) = \tan^{-1}\left(\frac{1}{s}\right)$ then $L\left(\frac{\sin at}{t}\right)$ is :

(A) $\frac{1}{a^2}\tan^{-1}\left(\frac{a}{s}\right)$

(B) $\frac{1}{a}\tan^{-1}\left(\frac{a}{s}\right)$

(C) $\tan^{-1}\left(\frac{a}{s}\right)$

(D) $a\tan^{-1}\left(\frac{a}{s}\right)$

49. Laplace transform of t sin at is :

(A) $\frac{1}{2}\frac{as}{\left(s^2+a^2\right)^2}$

(B) $\frac{as}{\left(s^2+a^2\right)^2}$

(C) $\frac{2as}{\left(s^2+a^2\right)^2}$

(D) $\frac{4as}{\left(s^2+a^2\right)^2}$

50. Laplace transform of $\left(e^{at}-\cos bt\right)/t$ is :

(A) $\ell n\frac{s-a}{\sqrt{s^2+b^2}}$

(B) $\ell n\left(\frac{\sqrt{s^2+b^2}}{s-a}\right)$

(C) $-\ell n\left(\frac{\sqrt{s^2+b^2}}{s-a}\right)$

(D) $-\ell n\left(\frac{s-a}{s^2+b^2}\right)$

51. Inverse transform of $\frac{3(s^2-2)^2}{2s^5}$ is :

(A) $\frac{3}{2}-3t^2+\frac{1}{4}t^4$

(B) $\frac{3}{2}+t^2-\frac{1}{4}t^4$

(C) $\frac{3}{2}+t^2+\frac{1}{4}t^4$

(D) $\frac{3}{2}-t^2-\frac{1}{4}t^4$

52. Solution of $(D^2 + DD' - 6D'^2)z = y\cos x$ is :

(A) $f_1(y + 3x) + f_2(y - 2x) + \sin x + y\cos x$

(B) $f_1(y - 3x) + f_2(y - 2x) + \sin x - y\cos x$

(C) $f_1(y + 3x) + f_2(y + 2x) + \sin x - y\cos x$

(D) $f_1(y - 3x) + f_2(y + 2x) + \sin x - y\cos x$

53. Inverse transform of $\frac{s}{\left(s^2+\omega^2\right)^2}$ is :

(A) $2\omega(t\sin\omega t)$

(B) $\omega(t\sin\omega t)$

(C) $\frac{1}{\omega}(t\sin\omega t)$

(D) $\frac{1}{2\omega}(t\sin\omega t)$

54. The Laplace transform of $te^{-t}\sin t$ is :

(A) $\dfrac{2s+2}{\left(s^2+2s+2\right)^2}$

(B) $\dfrac{2s+2}{\left(s^2-2s+2\right)^2}$

(C) $\dfrac{2s-2}{\left(s^2-2s+2\right)^2}$

(D) $\dfrac{2s+2}{\left(s^2-2s+2\right)^2}$

55. The solution of $y' + 3y = 10 \sin t$, $y(0) = 0$ is :

(A) $y = e^{3t} + \cos t + 3\sin t$

(B) $y = e^{-3t} - \cos t + 3\sin t$

(C) $y = e^{3t} - \cos t - 3\sin t$

(D) $y = e^{-3t} + \cos t - 3\sin t$

56. The Fourier transform of $f(x) = \begin{cases} e^x & \text{if } -a < x < a \\ 0 & \text{otherwise} \end{cases}$ is :

(A) $\dfrac{e^{a+ia\omega} + e^{-a+ia\omega}}{(1+i\omega)\sqrt{2\pi}}$

(B) $\dfrac{e^{-a+ia\omega} + e^{-a-ia\omega}}{(1+i\omega)\sqrt{2\pi}}$

(C) $\dfrac{e^{a-ia\omega} - e^{a+ia\omega}}{(1-i\omega)\sqrt{2\pi}}$

(D) $\dfrac{e^{a-ia\omega} - e^{-a+ia\omega}}{(1-i\omega)\sqrt{2\pi}}$

57. The solution of $xys = 1$ is :

(A) $z = \log x + \log y + \phi_1(x) + \phi_2(y)$

(B) $z = \log x - \log y + \phi_1(x) + \phi_2(y)$

(C) $z = \log x \log y + \phi_1(x) + \phi_2(y)$

(D) $z = \dfrac{\log x}{\log y} + \phi_1(x) + \phi_2(y)$

58. The solution of $x^2r + 2xys + y^2t = 0$ by Monge's method is :

(A) $z - \phi\left(\dfrac{x}{y}\right) = x\,\psi\left(\dfrac{x}{y}\right)$

(B) $z + \phi\left(\dfrac{y}{x}\right) = x\,\psi\left(\dfrac{y}{x}\right)$

(C) $z - \phi\left(\dfrac{y}{x}\right) = \psi\left(\dfrac{y}{x}\right)$

(D) $z + \phi\left(\dfrac{y}{x}\right) = \psi\left(\dfrac{y}{x}\right)$

59. The Fourier series of

$$f(x) = \begin{cases} 1 & \text{if } -\frac{\pi}{2} < x < \frac{\pi}{2} \\ -1 & \text{if } \frac{\pi}{2} < x < \frac{3\pi}{2} \end{cases}$$ is :

(A) $\frac{\pi}{4}(-\cos x + \frac{1}{3}\cos 3x - \frac{1}{5}\cos 5x + \cdots\cdots)$

(B) $\frac{\pi}{4}(\cos x + \frac{1}{3}\cos 3x + \frac{1}{5}\cos 5x + \cdots\cdots)$

(C) $\frac{4}{\pi}(\cos x - \frac{1}{3}\cos 3x + \frac{1}{5}\cos 5x - \cdots\cdots)$

(D) $\frac{4}{\pi}(\cos x + \frac{1}{3}\cos 3x + \frac{1}{5}\cos 5x + \cdots\cdots)$

60. The Fourier series of

$$f(x) = \begin{cases} 0 & \text{if } -2 < x < 0 \\ 1 & \text{if } 0 < x < 2 \end{cases}$$ is :

(A) $\frac{\pi}{2}\left(\sin\frac{\pi x}{2} + \frac{1}{3}\sin\frac{3\pi x}{2} + \frac{1}{5}\sin\frac{5\pi x}{2} + \cdots\cdots\right)$

(B) $\frac{1}{2} + \frac{2}{\pi}\left(\sin\frac{\pi x}{2} + \frac{1}{3}\sin\frac{3\pi x}{2} + \frac{1}{5}\sin\frac{5\pi x}{2} + \cdots\cdots\right)$

(C) $\frac{1}{2} - \frac{2}{\pi}\left(\sin\frac{\pi x}{2} + \frac{1}{3}\sin\frac{3\pi x}{2} + \frac{1}{5}\sin\frac{5\pi x}{2} + \cdots\cdots\right)$

(D) $-\frac{\pi}{2}\left(\sin\frac{\pi x}{2} + \frac{1}{3}\sin\frac{3\pi x}{2} + \frac{1}{5}\sin\frac{5\pi x}{2} + \cdots\cdots\right)$

61. The maximum value of $Z = 5x + 8y$

Subject to $3x + 2y \leq 36$

$x + 2y \leq 20$

$3x + 4y \leq 42$

and $x, y \geq 0$ is :

(A) 88

(B) 86

(C) 84

(D) 82

62. The minimum value of $Z = 4x_1 + x_2$

Subject to $3x_1 + x_2 = 3$

$4x_1 + 3x_2 \geq 6$

$x_1 + 2x_2 \leq 4$

and $x_1, x_2 \geq 0$ is :

(A) $\frac{20}{3}$

(B) $\frac{30}{7}$

(C) $\frac{18}{5}$

(D) $\frac{29}{5}$

63. The variable which represents amount by which solution value exceed a resource is :

(A) Slack variable

(B) Surplus variable

(C) Dependent variable

(D) Independent variable

64. A game with two players where the gain of one player equals the loss of the other is known as :

(A) Two person games

(B) Zero-sum game

(C) Two person zero sum game

(D) Non rectangular game

65. The value of the game whose pay off matrix is $\mathbf{A}\begin{bmatrix} 5 & 1 \\ 3 & 4 \end{bmatrix}$ ($\mathbf{B}$ above the columns) is given by :

(A) $\frac{17}{5}$

(B) $\frac{16}{5}$

(C) $\frac{19}{5}$

(D) $\frac{21}{5}$

66. The contrapositive of the statement "I come to class whenever there is going to be a quiz" is :

(A) If I do not come to class then there will be a quiz

(B) If I come to the class then there will be a quiz

(C) If there will not be a quiz then I do not come to class

(D) If I do not come to class then there will not be a quiz

67. If n is an integer and $3n + 4$ is odd, then :

(A) n is prime

(B) n is odd

(C) n is even

(D) n is a natural number

68. On the set of integers, the relation "greater than or equal" is :

(A) Only reflexive

(B) Only symmteric

(C) Only antisymmetric

(D) Only poset

69. By the principle of mathematical induction, $1^2 + 3^2 + 5^2 + \cdots + (2n-1)^2$ is :

(A) $\frac{n(n+1)(2n+1)}{6}$

(B) $\frac{n(2n-1)(2n+1)}{3}$

(C) $\frac{n(2n-1)(2n+1)}{6}$

(D) $\frac{n(n+1)(2n+1)}{3}$

70. A lattice is a partially order set $(L, \leq)$ in which every pair of elements $a, b \in L$ has :

(A) a greatest lower bound

(B) a least upper bound

(C) a glb and a lub

(D) a glb or a lub

71. In a distributive lattice $(L, \leq)$, if $b, c \in L$ and $b \wedge c = 0$ then :

(A) $c \leq b$

(B) $b \leq c$

(C) $c < b$

(D) $b < c$

72. The number of different Boolean function of degree n is :

(A) 2^{n-2}

(B) 2^{n-1}

(C) 2^n

(D) 2^{n+1}

73. If G = (V, E) be an undirected graph with e edges, then the sum of the degrees of vertices is :

(A) 5e

(B) 4e

(C) 3e

(D) 2e

74. In a simple graph, the maximum number of edges with n vertices is :

(A) $\frac{n(n+1)}{2}$

(B) $\frac{n(n-1)}{2}$

(C) $\frac{(n-1)(n-2)}{2}$

(D) $\frac{(n-2)(n-3)}{2}$

75. If a graph has 21 edges, 3 vertices of degree 4 and other vertices of degree 3, then the total number of vertices are :

(A) 13

(B) 14

(C) 15

(D) 16

76. If n is an integer and 3n + 2 is even, then n is :

(A) odd

(B) even

(C) prime

(D) perfect

77. A tree with 10 vertices has :

(A) 7 edges

(B) 8 edges

(C) 9 edges

(D) 10 edges

78. A full m-ary tree with i internal vertices contains :

(A) mi – 2 vertices

(B) mi – 1 vertices

(C) mi vertices

(D) mi + 1 vertices

79. The value of the Boolean function represented by $f(x, y, z) = xy + \bar{z}$ is :

(A) 1

(B) 0

(C) 2

(D) 3

80. If G be a connected planar simple graph with 'e' edges and 'v' vertices then the number of regions 'r' is :

(A) $-e+v-2$

(B) $e+v-2$

(C) $e-v+2$

(D) $e+v+2$

81. The derivative of $(iz^3 + 3z^2)^3$ at $2i$ is :

(A) 3

(B) 2

(C) 1

(D) 0

82. If a function $f(z) = u(x, y) + iv(x, y)$, then the Cauchy-Riemann conditions are :

(A) $u_x = v_y$ and $u_y = v_x$

(B) $u_x = -v_y$ and $u_y = v_x$

(C) $u_x = v_y$ and $u_y = -v_x$

(D) $-u_x = v_y$ and $u_y = -v_x$

83. If $f(z)$ is an analytic function then $\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)|f(z)|^2$ is :

(A) $|f'(z)|^2$

(B) $2|f'(z)|^2$

(C) $3|f'(z)|^2$

(D) $4|f'(z)|^2$

84. If $f(z)$ be a function which is analytic inside and on a simple closed curve C and z_0 be any point in the interior of C, then $\int_C \frac{f(z)}{z-z_0}dz$ is :

(A) $\pi i\, f(z_0)$

(B) $2\pi i\, f(z_0)$

(C) $3\pi i\, f(z_0)$

(D) $4\pi i\, f(z_0)$

85. The value of $\int_C \frac{dz}{z-3i}$ where C is the circle $|z| = \pi$ counter clockwise is :

(A) πi

(B) $2\pi i$

(C) $3\pi i$

(D) $4\pi i$

86. The residue at the pole of the function $\frac{e^z}{(z-\pi i)^3}$ is :

(A) $-\frac{1}{2}$

(B) $\frac{1}{2}$

(C) $\frac{1}{3}$

(D) $-\frac{1}{3}$

87. The residue at the pole of the function cot z is :

(A) -1

(B) -2

(C) 1

(D) 2

88. The value of the complex integration $\int_C \frac{3z-4}{z(z-1)(z-2)}dz$ where C is the circle $|z| = \frac{3}{2}$ is :

(A) πi

(B) $2\pi i$

(C) $-\pi i$

(D) $-2\pi i$

89. Using Cauchy's residue, the value of $\int_0^\infty \frac{dx}{x^2+1}$ is :

(A) $\frac{\pi}{4}$

(B) $\frac{\pi}{3}$

(C) $\frac{\pi}{2}$

(D) π

90. The function $Z\bar{z} + 1$ in the form $u(x, y) + iv(x, y)$ is :

(A) $(x^2 - y^2 + 1) + 0i$

(B) $(x^2 + y^2 - 1) + 2i$

(C) $(x^2 + y^2 + 1) + 0i$

(D) $(x^2 - y^2 - 1) + 3i$

91. Two normed linear spaces X and Y are said to be isometrically isomorphic if :

(A) there is a one to one correspondence between the elements of X and Y which makes the two spaces isomorphic

(B) there is a many to one correspondence between the elements of X and Y

(C) there is a one to many correspondence between the elements of X and Y

(D) there is a many to many correspondence between the elements of X and Y

92. The set C of all complex numbers is a complex linear space under :

(A) ordinary addition and subtraction

(B) ordinary multiplication and division

(C) ordinary addition and multiplication

(D) ordinary multiplication and subtraction

93. A Hilbert space is an inner-product space which is :

(A) finite dimensional and incomplete

(B) finite dimensional and complete

(C) infinite dimensional and incomplete

(D) infinite dimensional and complete

94. If x and y are any two vectors in a Hilbert space then :

(A) $|(x, y)| = \|x\| \|y\|$

(B) $|(x, y)| > \|x\| \|y\|$

(C) $|(x, y)| \leq \|x\| \|y\|$

(D) $|(x, y)| < \|x\| \|y\|$

95. The space ℓ_2 with the inner product of vectors $x = \{x_1, x_2, \cdots\cdots, x_n, \cdots\cdots\}$ and $y = \{y_1, y_2, \cdots\cdots, y_n, \cdots\cdots\}$ defined by $(x, y) = \sum_{n=1}^{\infty} x_n \bar{y}_n$ is :

(A) not convergent

(B) convergent

(C) divergent

(D) oscillating

96. If $\{e_i\}$ is an orthonormal set in a Hilbert space H, then for every vector x in H :

(A) $\sum |(x, e_i)|^2 \leq \|x\|^2$

(B) $\sum |(x, e_i)|^2 = \|x\|^2$

(C) $\sum |(x, e_i)|^2 \geq \|x\|^2$

(D) $\sum |(x, e_i)|^2 > \|x\|^2$

97. If $\{e_i\}$ is an orthonormal set in a Hilbert space H and if x is an arbitrary vector in H, then for each j :

(A) $x - \Sigma(x, e_i)e_i < e_j$

(B) $x - \Sigma(x, e_i)e_i$ is parallel to e_j

(C) $x - \Sigma(x, e_i)e_i = e_j$

(D) $x - \Sigma(x, e_i)e_i$ is perpendicular to e_j

98. Every non-zero Hilbert space contains :

(A) only orthonormal set

(B) an incomplete set

(C) a complete orthonormal set

(D) an incomplete orthonormal set

99. If N is a finite-dimensional normed linear space of dimension n, then the dimension of N^* is :

(A) $n - 2$

(B) $n - 1$

(C) n

(D) $n + 1$

100. If B is a Banach space and if B is reflexive then B^* is :

(A) reflexive

(B) symmetric

(C) antisymmetric

(D) transitive

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